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Making Months

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It is easy to take our calendar, with its system of weeks and months, for granted. But this system is really just a societal choice -- it has been debated and modified in the past. It will also have to be re-examined when humans colonize other planets. 'Making Months' illustrates a use of factoring, in a decision-making process.

Background: The Gregorian Calendar is based around the 365-day period it takes for seasons to return. For historical reasons we divide these 365 days unevenly into 12 months of 28 to 31 days. Because 7-day weeks were not important when the Romans developed this calendar, weeks seem to drift through the months and years.

The Gregorian Calendar has many pleasing features:

1 - The months and weeks include all 365 days. So every day has a weekday, a month, and a day number within that month.
2 - The weeks have 7 days, matching traditional Christian, Jewish, and Muslim religious practice.
3 - The months roughly match the lunar period of 29.53 days, the traditional meaning of "month".
4 - The year has exactly 12 months, a round number that divides nicely into halves and quarters.

But it also has some problems:

5 - There is not a whole number of weeks per month. So the first of January and the first of February are not the same day of the week,
6 - There is not a whole number of weeks per year. So some holidays, like the 4th of July or birthdays, fall on different days of the week each year. Other holidays, like Labor Day, stay the same day of the week but change date.

It is not possible to satisfy all listed criteria simultaneously. We cannot have 7-day weeks (criterion 2), a whole number of weeks per year (criterion 6), and all days included in weeks and months (criterion 1) because 7 is not a factor of 365.

Exercise: Propose reasonable calendar designs for an imaginary planet. Although you will not be able to meet all criteria at once, work towards these criteria:

1 - each day of the year is assigned to a specific week and month
2 - 7-day weeks (or 6 or 8 or 9 as assigned)
3 - months of 30 days (or 26 to 39 days as assigned)
4 - a number of months per year that divides evenly into quarters
5 - a whole number of weeks per month
6 - a whole number of weeks per year

Present 2 or 3 alternative designs, where each criterion is met by at least one of the designs. For each design show which criteria were met and which were missed.

Each student should receive a different combination of days per year (300-400), days per week (6-9), and days per month (26-39). The easiest pattern is to keep 7-day weeks and 30-day months while giving each student a year length of 300 + 9 * n (309, 318, 327...). You could give each planet a whimsical name like "Jeff-world" or "Planet Emma".

Presentation: First explain the exercise and assign numbers. Then present the Gregorian Calendar and the alternatives as an example of how the students are to do the assignment:

Exploring Alternatives 1: Factoring 365
The week problem would be solved most simply if the length of the week was a factor of 365. The prime factors of 365 are 5 and 73. We can have 5-day weeks, but then there is no uniform way to split our 73 weeks up into any number of months, since 73 is prime.

Exploring Alternatives 2: Factoring 364
What if we leave one day off the calendar? Factoring 364 = 2 * 2 * 7 * 13, so we could have 13 months of 28 days -- that's 4 7-day weeks. This is the "International Fixed Calendar" and is one of the most widely proposed designs. It would add a month named "Sol" between June and July. The extra day would be a holiday, as would leap day in leap years. Of course 13 months do not evenly divide into half-years or quarters. This calendar meets all criteria except 1 and 4.

The "World Calendar" would split the year into 4 quarters of 13 7-day weeks = 91 days each. Each quarter is then divided into three months, two with 30 days and one with 31 days. Each quarter always starts on a Sunday and there are 3 different possible arrangements of day-number with weekday -- 3 different calendar pages! January, April, July, and October would be 31-day months starting on Sundays. February, May, August, and November would be 30-day months starting on Wednesdays. March, June, September, and December would be 30-day months starting on Fridays. The extra day is a holiday called "World Day". This calendar meets all criteria except 1 and 5. But it does have a whole number of weeks per quarter, so it is better on criterion 5 than the Gregorian Calendar is.

Exploring Alternatives 3: Other Weeks
By most standards, 7 days is an inconvenient week length. 5- or 10-day weeks would be "round" in a decimal sense, 6- or 8-day weeks would be less of a change from 7-day weeks and have even numbers of days.

Dropping more days off the calendar gives us more options:

Days Dropped	Days Left	Factors	Possible Week Lengths
0	365	5 * 73	5
1	364	2 * 2 * 7 * 13	7
2	363	3 * 11 * 11	11
3	362	2 * 181	
4	361	19 * 19	
5	360	2 * 2 * 2 * 3 * 3 * 5	5,6,8,9,10
6	359	Prime	
7	358	2 * 179	

With a 5-days-off calendar, we get factors which can be combined in many different ways. Here are the ones I consider most appealing:

Months per Year	Weeks per Month	Days per Week
2 * 5 = 10	2 * 3 = 6	2 * 3 = 6
2 * 2 * 3 = 12	5	2 * 3 = 6
2 * 2 * 3 = 12	2 * 3 = 6	5
3 * 5 = 15	2 * 2 = 4	2 * 3 = 6

The ancient Egyptians had 12 30-day months and used their 5 extra days as festivals celebrating five gods.

The Maya had a calendar with 18 months of 20 days each. They considered the five extra days unlucky.

All of these calendars abandon criteria 1 and 2, though some meet all the other criteria.

Related Projects:

- Discuss how different calendars might affect school and work patterns. The length of the week would be most important. Would a 5-day week include one or two "weekend" days? What about 6-, 8-, 9-, and 10-day weeks? Would it be worth having shorter vacations to have longer "weekends"? Vice-versa? Schedules must add up to about 181 school-days per year.

- Have students tabulate all possible combinations of months-per-year, weeks-per-month, and days-per-week for the 360-day calendar.

Resources:

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