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Algebra Magic – Create Your Own Number Puzzles

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Many people think of algebra as dealing with solving equations that have letters and numbers, and that it is completely unrelated to anything outside the math classroom. Also, teachers have a difficult time motivating students to learn algebra. I have found that not only are number/magic puzzles quite common outside the classroom, but also that they motivate students, and that teachers can successfully use such number puzzles as an interesting lead-in to algebra. Here I discuss two number/magic puzzles that can be demystified and explained by using algebra.

Number Puzzle One

An old number trick has directions as follows (the numbers/letters in parentheses are algebraic equivalents of the steps):
Step 1: Write down the year you were born. [p]
Step 2: Write down a year when a memorable event in your life took place. [q]
Step 3: Write down your age as of the last day of this year. [y – p]
Step 4: Write down the number of years that elapsed since the memorable event (in Step 2) took place. [y – q]
Step 5: Add the 4 numbers obtained from Steps 1 through 4. [p + q + (y – p) + (y – q)=2y]

Prediction: Everyone who followed all the directions would have got the number 4006 (if the trick were carried out in the year 2003).
The prediction sounds magical, especially as everyone gets the same answer, even though all might have started off with different numbers.
So how does this work?

Explanation: Look at the variables that were added in Step 5. They are:

$$
\begin{aligned}
p + q + (y - p) + (y - q) &= p + q + y - p + y - q, \text{ by removing parentheses} \\
&= p - p + q - q + y + y, \text{ by rearranging the terms} \\
&= 2\,y, \text{ as the p's and q's cancel each other out} \\
&(= 4006, \text{ when } y = 2003)
\end{aligned}
$$

In other words, the number in Step 1 combined with that in Step 3, *deleted* the number in Step 1, giving the current year, 2003; similarly, the number in Step 2 combined with that in Step 4, *deleted* the number in Step 2, giving, once again, the current year, 2003. Hence, adding all the 4 numbers from Step 1 through Step 4 effectively deleted every number other than twice the current year, 2003, giving the result of 2 x 2003 = 4006! (Note that most number puzzles undo what has been done, so as to "force" a certain result, as can be easily proved by using deductive logic or by using algebra.)

Number Puzzle Two

This number puzzle gives the age of the person, and the number of siblings in the person's family (sample numbers and algebraic equivalents are given next to each step/direction).

Directions	Sample #s	Algebraic equivalent
1. Think of your age in years	15	$[10x+y]$
2. Multiply your age by 2	30	$2[10x+y]$
3. Add 10	40	$2[10x+y] + 10$
4. Multiply by 5	200	$5\{2[10x+y]+10\}=100x+10y+50$
5. Add # of siblings you have	203	$[100x+10y+50+z]$
6. Subtract 50	153	$[100x+10y+50+z]–50 = 100x+10y+z$

Predication: You are 15 years old, and you have 3 siblings.

Explanation: The first two digits on the left (15) give your age, and the rightmost digit (3) gives the number of siblings you have.

The algebraic equivalent of the steps result in Step 6, which is $100x + 10y + z$, where $z<10$. But $100x + 10y + z$ is the algebraic equivalent of a three-digit number, with the digits in the hundreds and tens places giving the age, and the digit in the ones place giving the number of persons in the house. (Note: the number of siblings has to be less than 10, otherwise this "trick" will not work. Why?)

Activity. Now, make up your own number puzzle, justify it algebraically, and test it out with your friends. Be careful about any cases where a number is "carried" from one decimal place to another. Puzzle Two avoided the "carrying" by restricting the number of siblings to less than 10.