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Why Is There a Tidal Bulge Opposite the Moon?

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Most everyone is aware that the Moon causes ocean tides on Earth, and coastal inhabitants usually have firsthand knowledge. A long day at the beach or a look through tide tables will demonstrate that there are usually two high tides or two low tides each day, and that they shift through time-of-day as the calendar advances.

There is a high tide on the Moon-side of Earth because of the Moon’s gravitational affect on Earth (and the ocean water), but why should there be a bulge of water on the opposite side of Earth, making two high tides separated by a little more than 12 hours every 24 hours?

**OBJECTIVE:** Use mathematics to understand tides and gravitation and how gravity works across astronomical distances.

**APPARATUS & PREPARATION:**

1. **Slinky**-type “lazy” spring, metal or plastic, and a hook to hang it from. The slinky should be short enough that it won’t touch the floor while hanging from a height great enough for students to see it from their seats. In advance of use, attach five “flags” (individually visible from the back of the classroom) that are separated by equal numbers of coils. The outermost flags (called sub-Moon water and antipodes water) can be separated by 1 or 2 coils from sub-Moon Earth and antipodes-Earth, respectively, and sub-Moon Earth and antipodes Earth can be separated by 15 coils each from the Earth center flag (In other words, the flags follow this symmetric pattern: sub-Moon water flag - 2 coils – sub-Moon Earth flag - 15 coils – Earth center flag - 15 coils – antipodes-Earth flag - 2 coils – antipodes water flag.) Feel free to experiment with all the separations. The outer flag pairs represent sea level and the solid Earth’s surface and the middle flag represents Earth’s center. The middle flag should be near the middle of the coil if possible, but high enough so that students at the back of the room can see all five flags above desk level.
2. **“Flags”** (5) to mark coils of the slinky spring. Stapled or paper-clipped or taped pieces of paper or thin cardboard can be folded around an individual slinky coil and fixed in place.
3. **Meter stick** or yardstick to measure the separations of the flags when the slinky is hanging without motion. The meter stick or a rod should be used to hang the slinky horizontally.

**PROCEDURE:**

Start by spreading the coil with both hands as it hangs on a rod held horizontally. Point out (or measure) the equal separations of the (sub-Moon water flag – sub-Moon Earth flag) and the (antipodes-Earth flag – antipodes water flag) pair and the equal separations of the (sub-Moon Earth flag – center flag) and the (center flag – antipodes Earth-flag) pair. The coil between the water flags represents Earth’s self-gravity. Letting go of the ends of the coil will prove the point as it collapses on itself.

Hang the coil vertically from the hook in front of the students and discuss what they are seeing now. They should note that the distances separating the flags all different magnitudes (even if one separation is matched to its horizontal separation). Separations of the water-Earth surface flag pairs at the top and bottom will be different, as will the separations of the Earth surface-center flag pairs. The upper two spacings are greater than the lower two spacings even though the number of coils is the same. Remind the students that the hanging coil, between the outer flags, now represents Earth and its oceans, under the influence of its own gravity (self-gravity) and the gravity of the Moon (the hook holding the coil up, *not* Earth’s gravity towards the floor). Ask for an explanation of the pattern of separation.

Answer: When stretched vertically (by hanging from the hook) an outside body (the hook and the slinky above the top flag) is applying a force on the slinky to hold it in place. Measuring the positions of the flags demonstrates that there is a different amount of force stretching the slinky along its length. This is analogous to the decrease in the force of gravity with distance from a mass. (Note we don’t feel this difference if we lift a hand-weight at sea level and again on top of a mountain or in an airplane. The change in distance from Earth’s center is just too little for our muscles to feel the difference, though it can be measured with instruments.)

**THE UNDERLYING PRINCIPLES:**

Tides may be defined as the DIFFERENCE of the gravitational attraction of an outside object THROUGH another (nearby) object.  In the Earth-Moon system, the strength of the Moon’s attraction is:

* Greatest on Earth’s surface directly below the Moon in the sky (Moon in the zenith), the sub-Moon point
* Weaker at Earth’s center and
* Weakest at the antipodes, the point on Earth’s surface on the line from Moon through Earth’s center to the far surface.

If one now considers the Earth falling towards the Moon, the sub-Moon point has a greater force on it than the center, which has a greater force on it than the antipodes.  So water (and the solid earth surface) on the Moon-side falls towards the Moon faster than the center which falls faster than the antipodes water (and solid earth surface there).  The antipodes water is being left behind by the center, which is being left behind by sub-Moon water.  Back on Earth, we see these as two bulges of water which we call tides (and we can measure solid earth tides as well).

It is important to understand that, while Earth’s gravity is stretching the slinky down, the important analogy is that the restoration force of the spring acting through the hook is the equivalent of the Moon’s gravity acting on Earth (and within the end flags, the spring represents Earth’s self-gravity with some lunar gravity). The restoration force along the hanging slinky changes (decreases closer to the floor) because a high coil (near the hook) has more coils and their weight beneath itself than does an individual coil lower down, which is holding fewer coils and weight beneath itself. Sketches of the gravitational force vectors involved (lunar and Earth self-gravity) at the sub-Moon point, Earth center, and antipodes may help illuminate the forces individually and their sums.

The spring analogy should not be pushed too far. Measurements made with the spring will not match the gravitational calculations in the DISCUSSION section, next. The restoration force for a spring is linear (not inverse-squared, as it is for gravity). It is given by F = -k x r, where k is the spring constant (different for lazy springs and stiff springs) and r is a measure of the extension. The minus sign indicates that the restoration force is in the direction opposite the extension.

**DISCUSSION:**

We want to compare the difference in the Moon’s pull on Earth’s center with its pull on the antipodes point (and the sub-Moon point). We can rewrite Newton’s Law of Gravitation, F = G x (m) x (m2)/r2 to give acceleration:

(1) Gravitational Acceleration = F/(m) = G x (m2)/r2, where G is the gravitational constant 6.674 x 10-17 (N km2/kg2) and (m2) is the mass of the Moon = 7.3490 x 1022 kg.

(2) Earth’s Center’s Gravitational Acceleration due to the Moon = 3.319 x 10-5 km/s2.

(3) Antipodes Gravitational Acceleration due to the Moon = 3.212 x 10-5 km/s2.  This is smaller than in (2) because the antipodes is one Earth radius (6378.14 km) farther than Earth’s center from the Moon, so the denominator is larger.

(4) The Earth Center-Antipodes difference in acceleration is 1.07 x 10-6 km/s2.

(5) The sub-Moon Point’s Gravitational Acceleration due to the Moon = 3.432 x 10-5 km/s2. This is larger than in (2) because the sub-Moon point is one Earth radius (6378.14 km) closer than Earth’s center to the Moon, so the denominator is smaller.

(6) The sub-Moon-Earth Center difference in acceleration is 1.13 x 10-6 km/s2.

The result in (4) is different from the result in (6) because the force of gravity decreases as the square of the distance. There is a calculable difference in the forces and in the differential forces at the sub-Moon, center, and antipodes points. The slinky illustrates that the water bulge is larger in the sub-Moon hemisphere and smaller in the antipodes hemisphere, reflecting both the calculations and reality.

**EXTENSION 1:**

Students can actually calculate solar tides, the difference in the force of gravity at these three points on the Sun-Earth line, based on the differences in their distances to the Sun’s center (Earth’s radius is 6378.14 km and the Sun’s distance is1.496x108 km).

Ocean and solid earth tides vary in magnitude because the Moon’s distance from Earth varies during its orbital revolution, the Sun’s distance from Earth varies during its orbital revolution, and the alignment of Sun and Moon varies.  Students can calculate these variations using values of periapsis (perigee of the Moon and perihelion of the Earth) and apoapsis (apogee of the Moon, aphelion of the Earth) if desired.

Periapsis of an ellipse = a x (1 + e) and

Apoapsis of an ellipse = a x (1 - e) where

a = the semi-major axis of the ellipse and

e = the eccentricity of the ellipse.

Both a and e are orbital elements (of Earth [to compute perihelion and aphelion distances] and of the Moon [to computer perigee and apogee distances]) that are easily found on the World Wide Web.

**EXTENSION 2:**

In the Solar System, the Moon and Earth trace different amplitude sinusoidal waves as their center of mass orbits the Sun smoothly. They revolve around each other much as an unbalanced dumbbell would spin: the light end makes a large circle while the heavy end makes a small circle about the dumbbell’s center of mass. (This last sentence is an oversimplification of the Earth-Moon system. See Edberg [2005] for a discussion and demonstration.)

Some sources explain that the water bulge on the antipodes hemisphere is due to the centrifugal force of the Earth (at the antipodes point) swinging around the Earth-Moon center of mass. It is not difficult to demonstrate that this effect is minor compared to the differential gravitational affects presented in the DISCUSSION above.

We can demonstrate this with Newton’s Laws:  His 2nd Law states that F = m x a and his Law of Gravitation states that F = G x (m) x (m2)/r2.

(7) Centrifugal force, while fictitious, can be computed with Fc = (m x v2)/r .

Equation (7) can be rewritten to define an acceleration (since one can assume that the same mass of water will be used in this comparison):

(8) Centrifugal Acceleration ac = Fc/m = (v2)/r .

The velocity, v, of the antipodes point is calculated from the circumference of its orbit divided by the duration of the orbit.

(9) Radius of antipodes orbit = (2 x (Earth’s radius) – (CM offset from surface)).

Earth’s equatorial radius is 6378.14 km and the Nova website gives the CM offset as 1070 mi = 1722 km below Earth’s surface.

(10) Radius of antipodes orbit = 11034.28 km

(11) Circumference of antipodes orbit = (2 x pi x 11034.28) = 69330.44 km

The Moon and Earth orbit the CM in 27.322 days (in an inertial system fixed on quasars, not based on the Sun). There are 86400 seconds in a day so the velocity is

(12) v = 69330.44/(27.322 x 86400) = 0.0294 km/s

Using (8), the acceleration due to the centrifugal force is

(13) ac = ((0.0294)2)/11034.28 = 7.8 x 10-8 km/s2.

Recall DISCUSSION calculation (4). The difference in Earth Center-Antipodes acceleration is 1.07 x 10-6 km/s2, which is almost 14 times greater than the acceleration due to CM centrifugal force.

From DISCUSSION calculation (6) the difference in acceleration sub-Moon-Earth Center is 1.13 x 10-6 km/s2, which is 14.5 times greater than the acceleration due to CM centrifugal force.

One can conclude that differential lunar gravity, the Moon’s tide, is the cause of 92.7% of the antipodes water bulge.

**FOR MORE INFORMATION:**

Tidal Curiosities: <http://www.pbs.org/wgbh/nova/venice/tide_curiosities.html>

**REFERENCES:**

Edberg, S. J., 2005, *The Moon Orbits the Sun?!?!*, PUMAS, <https://pumas.gsfc.nasa.gov/examples/index.php?id=86>

Constants, masses, and orbital values came from the *Observer’s Handbook 2011* of the Royal Astronomical Society of Canada, which also includes a chapter on tides.  Conversion to km from m followed majority rule among the units needed in the calculations.

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All calculations were made on an ancient HP67 calculator [RPN: ENTER > = ].

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