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Rotational Motion and Rocket Launches

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Prerequisites: Beginning circle geometry, estimation, simple fractions. Beginning trigonometry would be helpful.

Calculations: If calculators are used, consider 3.14 to be an adequate estimate of Pi. Otherwise, 3 will do just fine. The answers given are calculated with 10 significant digits of Pi. Your students' answers may vary slightly from the solutions for this reason.

Recommended props: A globe
A table of latitudes of major world cities

Rotation: My friend Kyle works for Boeing on the Sea-Launch project. Sea-Launch is a portable, floating rocket launch pad for putting satellites into space. So, if Korea wants Sea-Launch to put a new digital TV satellite into space, and Sea-Launch can go anywhere in the world with water, where should Kyle take Sea-Launch? (No, it doesn't have a steering wheel. It gets pulled by tugboats.)

Well, like any scientist, he scratches his head for a minute, then works it out. Grab a pencil and follow along!

We start with what we know: **The Earth is round**. What do we know about round things?

Nature gives us a magic number, Pi. The circumference of a circle is Pi times its diameter. So, if you were to walk around a merry-go-round whose diameter is 10 feet, you would have walked Pi * 10 feet, or about 31 feet. Pi is approximately 3.14159...; since it's an irrational number, we don't (and never will) know all the digits.

What else do we know? **The Earth spins**. What do we know about spinning things?

1. Imagine you're standing on the edge of this merry-go-round. Every time it rotates, you've moved Pi * 10 feet.

Warm-up question: If it rotates once a second, how far do you move each second?
Answer: Pi * 10 feet, or about 31 feet.

Follow-up: If it rotates twice a second, how far do you move each second?
Answer: 2 * Pi * 10 feet, or about 63 feet.
How about if it rotates three times a second? Four? Five and a half? You'd get dizzy.
Answers: 94 ft, 126 ft, 173 ft

2. Let's go back to moving at two rotations per second. **Speed** is the distance you travel divided by the amount of time you were moving. How fast are you moving? Give your answer in feet per second. Answer: 63 feet / 1 second = 63 ft/sec

3. Now, double the size of the merry-go-round, so that it's 20 feet across. If it rotates once per second, how fast will you be moving? What if the merry-go-round rotates once every two seconds? Answers: 63 ft/sec (the same as answer #2!), 31 ft/sec

4. Suppose you're pushing the merry-go-round. Most people can run at a speed of about 10 feet per second. If you're running that fast, holding on to the edge of the 10-foot merry-go-round, how long will it take you to run in a complete circle? This is called **the period of the rotation**.
Answer: (31 feet) / (10 ft/sec) = 3.1 seconds

5. How many times will the merry-go-round rotate per second? This should be less than once per second. This is **the frequency of the rotation**. Note: frequency * period = 1.
Answer: frequency = (1/period) = (1 / 3.1 sec) = 0.32 (1/sec) = 0.32 Hz (Hertz = cycles/sec)

That was fun. Let's play around to get a feel for the numbers:

6a. If you double the frequency of rotation, what happens to the period? Answer: It halves

6b. If you double how fast you can run, what happens to the period? Answer: It halves

6c. If you double the size of the merry-go-round, what happens to the period? Answer: It doubles

7. If you make the merry-go-round 5% bigger, how much faster do you have to run in order to turn it with the same period? Answer: 5% faster

8. How does your answer to question 7 depend on the size of the merry-go-round?
Answer: It doesn't. The 5% change in diameter leads to a 5% change in circumference.

Advanced: This is what is known as a "**linear system**," because if you plot the period as a function of the diameter, you get a straight line.

The Punchline: The Earth (a large merry-go-round) has a diameter of 8000 miles and rotates once per day. It has a rotation period of 24 hours. If you were at the equator, you'd move at about 1047 miles per hour around the center of the planet. Why?
Answer: Pi * 8000 mi / 24 hours = 1047.2 mph.

9. If you were three miles above sea level on a mountain near Quito, Equador, how much faster would you move than you would at sea level in the Galapagos Islands? (Hint: What is the Earth's radius?) Answer: remembering problem 7, the answer is 4003/4000, 1.00075, or .075%.

Alternate: The speed on the mountain is Pi * 8006 miles / 24 hours = 1047.983 mph. The speed at sea level is Pi * 8000 miles / 24 hours = 1047.198 mph. The difference is .79 mph. This answer is correct, but not necessarily as versatile, since it depends on which approximation you use for Pi. The first answer didn't depend on Pi.

A common error will be to confuse radius and diameter, leading to answers such as 8003/8000. If this is a problem, suggest that the student draw two concentric circles -- one representing sea level, and the other a mountain range -- and ask the student to explain how the picture might apply to this problem.

10a. If you were to fly around the world at the equator, at 6 miles above sea level, how far would you have to fly? Answer: Pi * 8012 mi = 25170 mi

10b. In 1993-1994, a group of explorers attempted to drive from London to New York (see http://tdv.com/html/richard_c.html). They actually had to do some of the trip on foot when one their vehicles sank in the Bering Sea. Because they did most of the driving near the arctic circle, they didn't have to drive 25000 miles like they would have going around the equator. If you were to drive/walk/sail around the world along the arctic circle, how far would you have to go? (Hint: look at a globe.) Answer: If you are confronted with blank stares from your students, suggest to them that they use this formula:

Diameter a circle around the Earth = Diameter of Earth * Cos(latitude).

Their calculators probably have a Cos button. Otherwise, Cos(60) = 1/2.

10c. Suppose you're at Boeing headquarters in Seattle, at latitude 47. How fast is the Earth spinning there? (Hint: The cosine of 47 degrees is 0.682)

Answer: 1047 mph * Cos(47) = 714 mph.
Alternate: Pi * 8000 miles * Cos(47) / 24 hours = 714 mph

You could use latitude 45 instead, since Cos(45) = 1/Sqrt(2) is a "nice" number and gives you a similar answer, 740 mph.

Now, put it all together:

11a. In order to launch a rocket, you have to get it going pretty fast. And rocket fuel is expensive. Therefore, rockets always launched towards the East. Why?

Answer: So that the rocket can get a speed boost from the Earth's rotation.

11b. Why does NASA launch rockets from Kennedy Space Center in Florida (latitude ~ 29˚), instead of from, say, NASA's Moffett field in California (latitude ~ 37.5˚)?

Answer: Because Florida is as close to the equator as you can easily get in this country. The extra boost from the Earth's rotation saves millions of dollars in rocket fuel! Also, it's so that they can launch over the ocean. Rockets sometimes blow up, and you don't want the pieces to land on anyone.

Corollary: If a satellite costs $500,000,000, and 2% of rocket launches blow up, how much should you pay for insurance on a satellite launch? Answer: $10 million + administration fees.

12. This should be easy: If Sea-Launch is in the Pacific Ocean, where would be a good place for Kyle to launch the Korean TV satellite?

Answer: Anywhere near the equator in the Pacific Ocean. You would also want to be away from South America and any inhabited islands.